DOCUMENT RESUME

ED 022 932 08 VT 006 886

By-Rahmlow, Harold F.; And Others

OCCUPATIONAL MATHEMATICS; EQUIVALENT FORMS. REPORT NO. 16-C, BOOKLET II. FINAL REPORT.

Washington State Coordinating Council for Occupational Education, Olympia.; Washington State Univ., Pullman. Dept. of Education.
Spons Agency-Office of Education (DHEW), Washington, D.C.
Bureau No-BR-7-0031
Pub Date Jun 68
Grant-OEG-4-7-070031-1626
Note-126p.
EDRS Price MF-$0.50 HC-$5.12
Descriptors-*ARITHMETIC, *FRACTIONS, FUNDAMENTAL CONCEPTS, *NUMBERS, *PROGRAMED TEXTS, *VOCATIONAL EDUCATION

This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to: (1) change integers into equivalent forms, (2) change fractions into equivalent forms, (3) recognize prime numbers up to 20, (4) factor the number 100 into primes, and (5) reduce literal or numeric fractions. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 882-VT 006 909, and VT 006 975. (EM)

BOOKLET II

OF

Report No. 16-C

Occupational Mathematics

EQUIVALENT FORMS

The numbers 1, 2, 3, 4 ... that we have been working with are called integers. You may have noticed that some of the integers are made up of other integers that can be factored or "broken down." For example, 4 can be factored into 2 x 2. 6 can be factored to 3 x 2, 8 can be factored to 4 x 2 but 4 can then be factored into 2 x 2, so 8 = 2 x 2 x 2, etc.

What about the integers 2, 3, 5, 7, etc? They cannot be factored further. 5 can only be written as 5 x 1 which are non-factorable to begin with. The integers that cannot be factored or written in any other form other than itself and 1 are called <u>prime numbers</u>. Such integers as 2, 3, 5, 7, 11 are primes.

NOTE: The number 1 is not considered a prime number.

Which of the following sets of numbers are all "primes?"

(a) 2, 5, 13, 15 Turn to page 116

(b) 3, 7, 9, 11 Turn to page 119

(c) 5, 11, 17, 19 Turn to page 117

Correct!

Numbers which cannot be written or factored into any other form than itself and 1 are primes.

Which one of the following groups are all primes?

(a)	1, 7, 2, 11	Turn to page 120
(b)	2, 7, 9, 13	Turn to page 122
(c)	Neither (a) nor (b)	Turn to page 115

Correct! Neither (a) nor (b) contains only primes.

In (a) 1 is not the prime. Because of our definition of primes as being a non-factorable number which can only be written as itself and 1, we must throw out 1. The first prime is 2. Also, in (b) 9 is not a prime as it can be written 9 = 3 x 3.

The primes from 1 to 20 are:

(a) 2, 3, 5, 7, 11, 12, 13, 17, 19
Turn to page 128

(b) 1, 2, 3, 5, 7, 11, 13, 17, 19
Turn to page 120

(c) 2, 3, 5, 7, 11, 13, 17, 19
Turn to page 118

Your answer was (a) 2, 5, 13, 15. Incorrect!

2, 5, 13 are primes; however, 15 can be reduced to
$15 = 5 \times 3$.

Remember, a prime is a number which cannot be factored into integers except itself and 1.

Look at the following integers and determine which group contains all prime numbers.

(a) 7, 13, 15, 19 Turn to page 121

(b) 2, 17, 3, 5 Turn to page 114

(c) 3, 13, 10, 5 Turn to page 121

Correct! 5, 11, 17, 19 are all prime numbers which cannot be written in any other form except itself and 1.

Which of the following are primes?

(a) 1, 7, 2, 11 Turn to page 120

(b) 2, 7, 9, 13 Turn to page 126

(c) Neither (a) nor (b) Turn to page 115

Very good! 2, 3, 5, 7, 11, 13, 17, 19 are the only primes from 1 to 20.

Turn to page 133.

Your answer was (b) 3, 7, 9, 11. Incorrect!

3, 7, 11 are primes; however, 9 can be rewritten as
$9 = 3 \times 3$.

Remember a prime is a number which cannot be factored into integers except itself and 1.

Look at the following integers and determine which group contains all prime numbers.

(a) 7, 13, 15, 19 Turn to page 121

(b) 2, 17, 3, 5 Turn to page 114

(c) 3, 13, 10, 5 Turn to page 121

Incorrect!

We said that a prime is a number which cannot be written in any other form besides itself and 1. Because of our definition, we must exclude 1. Therefore, 2 is the first prime.

Which group below contains only prime numbers?

(a) 1/3, 1/7, 2, 7 Turn to page 123

(b) 1/3, 1/6, 11, 13 Turn to page 125

(c) 19, 17, 13, 11 Turn to page 127

You are having trouble understanding what a prime is.

Ask your human teacher for help and then return to page 113.

Your answer was incorrect.

You are having your ups and downs.

Seek help from your human teacher and then return to page 113.

Wait a minute! Think about the definition of a prime number. Does it include fractions? NO!

Return to page 113 and reread the definition carefully this time. Then start again.

Your answer was incorrect!

You are having your ups and downs.

Seek help from your human teacher and then return to page 113.

Wait a minute! Think about the definition of a prime number. Does it include fractions? NO!

Return to page 113 and reread the definition carefully this time. Then start again.

You don't seem to be sure what a prime number is.

Return to page 113 and reread the definition then continue working.

Your answer of 19, 17, 13, 11 is correct! They are all primes.

Try one more.

Which of the following are primes?

(a) 1, 7, 2, 11 Turn to page 120

(b) 2, 7, 9, 13 Turn to page 124

(c) Neither (a) nor (b) Turn to page 115

Incorrect!

12 is not a prime. However, it is a number composed of primes: 12 = 6 x 2 which is 12 = (3 x 2) x 2 or 12 = 3 x 2 x 2.

Are 1/3, 1/7, 1/11, 2, 3 primes?

(a) Yes Turn to page 130

(b) No Turn to page 131

Correct! 2, 3, 5, 7, 11, 13, 17, 19 are all the primes from 1 to 20.

Now that you understand what prime numbers are turn to page 133 and we'll see how they are used.

Your answer said that 1/3, 1/7, 1/11, 2, 3 are primes.

Now wait a minute. 2 and 3 are primes but who said anything about fractions being primes. They are not!

Return to page 113 and read the definition of a prime number again. Then continue working from there.

Your answer was (b) no. Very good!

2,3 are primes but fractions are not.

Are 11, 7, 19, 17, 13, 3, 2, 5 all the prime numbers from 1 to 20?

(a) Yes Turn to page 129

(b) No Turn to page 132

Your answer was (b) no. Incorrect! Let's look at the numbers from 1 to 20.

1	Not prime by definition	11	Prime
2	Prime	12	= 3 x 2 x 2
3	Prime	13	Prime
4	= 2 x 2	14	= 7 x 2
5	Prime	15	= 5 x 3
6	= 3 x 2	16	= 2 x 2 x 2 x 2
7	Prime	17	Prime
8	= 2 x 2 x 2	18	= 3 x 3 x 2
9	= 3 x 3	19	Prime
10	= 5 x 2	20	= 2 x 2 x 5

Do you see where you made your mistake? Good! Let's go to page 114 and continue.

Now that we have established what a prime number is, what good is it? Well, if you'll recall earlier when we were reducing fractions such as $\frac{3 \times 2}{2}$ we cancelled the two's giving us $3 \times 1 = 3$.

This was pretty straight forward. What if we wanted to reduce a fraction like $\frac{3 \times 9}{12}$? By the methods we used earlier, we would be unable to reduce this fraction. However, by using prime numbers we can easily reduce this fraction. How? First of all we have to see that you can write integers in terms of primes. For example, $9 = 3 \times 3$, the 3's being prime numbers. Also, $10 = 5 \times 2$, the 5 and 2 being prime numbers. But what about a number like 12? Well, $12 = 6 \times 2$, correct? But 6 is not a prime. Therefore, we must reduce it to 3×2. Now, we have $12 = 6 \times 2$ which equals $12 = (3 \times 2) \times 2$.

or

$12 = 4 \times 3$ and $12 = (2 \times 2) \times 3$. So, 12 is composed of the prime integers 2, 2, 3.

What prime integers is 8 composed of?

(a) 4×2 Turn to page 139

(b) $2 \times 2 \times 2$ Turn to page 138

Your answer was incorrect!

You are having trouble recognizing prime numbers.

Get help from your human teacher and then return to the section on primes, page 113.

Your answer was (a) 5 x 2 x 2. Good, that is correct!

Try another one.

The integer 23 is composed of what primes?

(a) 23 x 1 Turn to page 120

(b) 11 x 3 Turn to page 137

(c) None of the above, It is prime.
Turn to page 140

Incorrect!

Remember we are looking for the prime numbers.

9 is not a prime as it can be broken down to 3 x 3.

Therefore, 18 = 3 x 3 x 2.

Try this one.

The prime number members of 28 are:

(a) 2 x 7 x 2 Turn to page 143

(b) 14 x 2 Turn to page 147

Incorrect! (b) cannot be true as 11 x 3 = 33 not 23.

Shame on you!

Here's a chance to redeem yourself.

The prime number members of 28 are:

(a) 2 x 7 x 2 Turn to page 143

(b) 14 x 2 Turn to page 134

Very good! $8 = 2 \times 2 \times 2$ which are all primes.

What prime integers is 18 composed of?

(a) 6×3 Turn to page 145

(b) 9×2 Turn to page 136

(c) $3 \times 3 \times 2$ Turn to page 142

Although it is true that 8 = 4 x 2, 4 is not a prime.

4 can be reduced to 2 x 2. Therefore, 8 = 2 x 2 x 2.

Remember, we want all the parts to be prime numbers.

Question:

What prime integers is 20 composed of?

(a) 5 x 2 x 2 Turn to page 135

(b) 4 x 5 Turn to page 141

(c) 10 x 2 Turn to page 141

Very good! 23 is a prime number.

Try this one.

What primes compose 18?

(a) 6 x 3 Turn to page 145

(b) 9 x 2 Turn to page 136

(c) 3 x 3 x 2 Turn to page 142

Your answer is incorrect!

You seem to be having trouble recognizing prime numbers.

See your human teacher and then return to page 113,

and review primes.

Excellent! 18 is composed of the prime integers 3, 3, 2.

You are catching on very well. Keep it up.

Turn to page 144.

Correct! The primes in 28 are 2 x 7 x 2.

Here's another one.

What prime integers compose 8?

(a) 4 x 2 Turn to page 147

(b) 2 x 2 x 2 Turn to page 138

This idea of writing integers in terms of primes such as 18 = 3 x 3 x 2 where 3, 3, 2 are primes is called <u>factoring</u>.

We say that we factored the integer 18 into prime integers.

Factoring the integer 16 into primes, we obtain:

(a) 4 x 4 Turn to page 148

(b) 8 x 2 Turn to page 148

(c) 4 x 2 x 2 Turn to page 151

(d) 2 x 2 x 2 x 2 Turn to page 150

Incorrect!

Remember, we are looking for the <u>prime</u> members.

6 is not a prime as it can be broken down to 3 x 2.

Therefore, 18 = 3 x 3 x 2.

Try this one.

What prime integers is 20 composed of?

(a) 5 x 2 x 2 Turn to page 135

(b) 4 x 5 Turn to page 141

(c) 10 x 2 Turn to page 141

Excellent! 36 does factor into the primes 3 x 2 x 3 x 2 or 3 x 3 x 2 x 2 which are the same.

Now that you can factor integers into their primes, we can reduce such fractions as $\frac{3 \times 9}{12}$.

Turn to page 158 and keep up the good work.

You are sure having your ups and downs.

Tell your human teacher where you are. Then return to the section on primes, page 113.

Your answer was either (a) 4 x 4 or (b) 8 x 2.

Incorrect!

4 x 4 does equal 16, but 4 is not prime.

8 x 2 does equal 16, but 8 is not prime.

Don't let the word factoring throw you. We are doing the same process as before. That of breaking the integer down into its lowest parts.

Can the integer 13 be factored into primes?

(a) Yes Turn to page 149

(b) No Turn to page 155

Incorrect! The number 13 cannot be factored into primes.

You are still having trouble recognizing primes.

Get help from your teacher and then return to the section on primes, page 113.

Very good! 16 factors into 2 x 2 x 2 x 2.

Try one more.

What are the prime factors of 36?

(a) 9 x 2 x 2 Turn to page 154

(b) 3 x 2 x 3 x 2 Turn to page 146

(c) 3 x 3 x 2 x 2 Turn to page 146

No! 4 x 2 x 2 are not the primes of 16.

They are factors of 16 but not the prime factors.

Remember, when we factor an integer we want the prime parts.

Can the integer 13 be factored into primes?

(a) Yes Turn to page 149

(b) No Turn to page 155

Correct! 2 x 2 x 2 x 3 are the prime factors of 24.

Try another one.

What are the prime factors of 36?

(a) 9 x 2 x 2 Turn to page 154

(b) 3 x 2 x 3 x 2 Turn to page 146

(c) 3 x 3 x 2 x 2 Turn to page 146

You are still having trouble recognizing prime numbers.

Return to the section on primes, page 113, after you get help from your teacher.

Incorrect! 9 x 2 x 2 are not the prime factors of 36. 9 is not prime, it can be factored to 3 x 3.

Question:

The prime factors of 30 are 5, 2, 3.

(a) Yes Turn to page 156

(b) No Turn to page 153

Correct! The integer 13 cannot be factored.

Try this one.

What are the prime factors of 24?

(a) 2 x 3 x 4 Turn to page 153

(b) 2 x 2 x 2 x 3 Turn to page 152

The prime factors of 30 are indeed 5, 2, 3.

Try one more.

The prime factors of 27 are:

(a) 3 x 3 x 9 Turn to page 147

(b) 3 x 3 x 3 Turn to page 157

(c) 9 x 3 Turn to page 153

Correct! The prime factors of 27 are 3 x 3 x 3.

Now that you can factor integers into their primes we can reduce such fractions as $\frac{3 \times 9}{12}$.

Turn to page 158 and keep up the good work.

Now that we know how to factor integers into primes we can apply this to reducing fractions such as $\frac{3 \times 9}{12}$.

The way in which we do this is to factor the integers in the numerator and the denominator to their primes. After we do this, we can cancel out the quantities and get our reduced fraction. Let's take a look at an example.

We want to reduce the fraction $\frac{4 \times 3}{2}$.

First we find the prime factors of each integer.

$4 = 2 \times 2$, 3 is already prime, and in the denominator 2 is already prime.

Now, we replace the 4 by its prime factors, 2×2.

This gives us $\frac{(2 \times 2) \times 3}{2}$. Now, we can cancel one pair of 2's.

Thus, $\frac{\cancel{2} \times 2 \times 3}{\cancel{2}}$ leaves us with $2 \times 3 = 6$.

Therefore, the fraction $\frac{4 \times 3}{2} = 6$.

Do you think you can do one on your own?

(a) Yes Turn to page 160

(b) No Turn to page 162

Excellent! 1/3 is the correct answer since

$$\frac{4}{12} = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 3} = \frac{1}{3}$$

Try one more.

3/12 will reduce to:

(a) 1/4 Turn to page 168

(b) 3/4 Turn to page 166

Your answer was (a) yes.

Okay, try this one.

Reduce the fraction $\frac{6 \times 3}{4}$. The fraction will be:

(a) $\frac{3 \times 2 \times 3}{4} = \frac{18}{4}$ Turn to page 167

(b) $\frac{3 \times \cancel{2} \times 3}{\cancel{2} \times 2} = \frac{9}{2}$ Turn to page 171

Excellent! The fraction $\frac{24}{14} = \frac{12}{7}$.

Reduce the fraction $\frac{17}{14}$.

(a) 3/2 Turn to page 172

(b) 17/14 Turn to page 180

(c) Doesn't Reduce Turn to page 180

Your answer was (b) No.

Don't feel bad, it does look complicated at first.

Possibly another example will help.

Say we want to reduce the fraction 9/6.

1. The first thing we must do is find the prime factors.

   For 9 - - - - - - - - - we have 3 x 3

   For 6 - - - - - - - - - we have 3 x 2

2. Now the second step is to substitute these prime factors into the fraction.

   For 9 - - - - - - - - - we substitute 3 x 3

   For 6 - - - - - - - - - we substitute 3 x 2

3. We now have $\frac{3 \times 3}{3 \times 2}$

4. We can now cancel a pair of 3's as 3/3 = 1.

   This looks like $\frac{\cancel{3} \times 3}{\cancel{3} \times 2}$.

5. This leaves us with the fraction 3/2 which cannot be reduced lower as they are primes.

Okay, now you try this one.

In reducing the fraction 4/12 we first:

(a) change it to $\frac{2 \times 2}{4 \times 3}$ Turn to page 179

(b) change it to $\frac{2 \times 2}{2 \times 2 \times 3}$ Turn to page 165

Excellent! $\frac{8 \times 6}{18} = \frac{2 \times 2 \times 2 \times 3 \times 2}{3 \times 3 \times 2}$.

Now that you can factor the numerator and denominator into primes, let's apply it to reducing fractions.

Reduce the fraction 4/12.

(a) $\frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 3} = \frac{1}{3}$ Turn to page 159

(b) $\frac{\cancel{2} \times 2}{\cancel{2} \times 6} = \frac{2}{6}$ Turn to page 166

(c) $\frac{4}{2 \times 6} = \frac{4}{12}$ Turn to page 164

Your answer was incorrect!

You seem to be having trouble factoring integers into primes.

See your human teacher for help and return to the section on factoring integers to primes on page 133.

Very good! We first change it to prime factors.

Thus, $\frac{4}{12}$ becomes $\frac{2 \times 2}{2 \times 2 \times 3}$.

Now, we must cancel the like terms. It will look like:

(a) $\frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 3} = \frac{1}{3}$ Turn to page 159

(b) $\frac{\cancel{2} \times 2}{\cancel{2} \times 2 \times 3} = \frac{2}{6}$ Turn to page 190

Your answer is incorrect!

Be more careful this time. Remember to factor the integers to primes and then cancel completely.

Factor and reduce this fraction: 15/6

(a) 5/2 Turn to page 198

(b) 3/2 Turn to page 195

(c) 5/3 Turn to page 191

Your answer was incorrect! Let's see how the problem is done.

We want to reduce the fraction $\frac{6 \times 3}{4}$. We must first find the prime factors of the integers.

The prime factors of 6 are 3 x 2

3 is already prime so we leave it.

Thus, on the top (numerator) we have 3 x 2 x 3.

On the bottom (denominator) we find the prime factors of 4 which are 2 x 2.

Thus, we have $\frac{3 \times 2 \times 3}{2 \times 2}$.

Let's see if you can factor the numerator and denominator for the fraction 8/6.

(a) $\frac{4 \times 2}{3 \times 2}$ Turn to page 164

(b) $\frac{2 \times 2 \times 2}{6}$ Turn to page 164

(c) $\frac{2 \times 2 \times 2}{3 \times 2}$ Turn to page 174

Correct! 3/12 reduces to $\frac{\cancel{3}}{\cancel{3} \times 2 \times 2} = \frac{1}{4}$

Now that you can reduce fractions of the type 3/12, turn to page 175 and let's try a little harder type.

Very good! The fraction $\frac{6 \times 9}{12}$ can either be written as $\frac{2 \times 3 \times 3 \times 3}{2 \times 2 \times 3}$ or $\frac{3 \times 2 \times 3 \times 3}{2 \times 3 \times 2}$. The order makes no difference.

Now that you can factor the numerator and denominator into primes, let's carry it a step farther and cancel the like terms and then reduce the fraction.

Reduce the fraction 4/12.

(a) $\frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 3} = \frac{1}{3}$ Turn to page 159

(b) $\frac{\cancel{2} \times 2}{\cancel{2} \times 2 \times 3} = \frac{2}{6}$ Turn to page 166

Correct! $\frac{18}{12} = \frac{\cancel{3} \times 3 \times \cancel{2}}{\cancel{3} \times 2 \times \cancel{2}} = \frac{3}{2}$

The fraction 17/14 reduces to:

(a) 3/2 Turn to page 196

(b) 17/14 Turn to page 180

(c) Doesn't reduce Turn to page 180

Good! You saw that we must factor the integers into their primes and cancel getting our answer of 9/2.

Reduce the fraction 24/14.

(a) 12/7 Turn to page 161

(b) 3/2 Turn to page 181

(c) 7/12 Turn to page 176

Incorrect! The fraction 17/14 does not reduce.

The numerator (top) is a prime and 14 is 7 x 2.

Thus, this fraction cannot be reduced.

Does the fraction 11/8 reduce?

(a) Yes Turn to page 183

(b) No Turn to page 177

Good! We must first change it to $\frac{3 \times 2}{3 \times 3}$.

Try one more.

Factor the fraction $\frac{6 \times 9}{12}$ into primes.

(a) $\frac{2 \times 3 \times 3 \times 3}{2 \times 2 \times 3}$ Turn to page 169

(b) $\frac{3 \times 2 \times 3 \times 3}{2 \times 3 \times 2}$ Turn to page 169

(c) $\frac{3 \times 2 \times 9}{4 \times 3}$ Turn to page 164

Your answer was (c) $\frac{2 \times 2 \times 2}{3 \times 2}$. Correct.

Try one more.

Factor the numerator (top) and denominator (bottom) of the following fraction into primes: $\frac{8 \times 6}{18}$

(a) $\frac{4 \times 3 \times 2}{9 \times 2}$ Turn to page 181

(b) $\frac{2 \times 2 \times 2 \times 3 \times 2}{3 \times 3 \times 2}$ Turn to page 163

Now we will reduce fractions of a more complicated nature. Remember the same principles apply--first we find the prime factors of the integers and then cancel.

What does the fraction $\frac{12}{16}$ reduce to?

(a) $\frac{2 \times 3 \times \cancel{2}}{8 \times \cancel{2}} = \frac{6}{8}$ Turn to page 186

(b) $\frac{\cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times 2 \times \cancel{2}} = \frac{3}{4}$ Turn to page 178

(c) $\frac{\cancel{2} \times 6}{\cancel{2} \times 4 \times 2} = \frac{6}{8}$ Turn to page 186

Whoa! Let's look again.

$24 = 8 \times 3 = 2 \times 2 \times 2 \times 3$

$14 = 7 \times 2$

Thus, $\dfrac{2 \times 2 \times \cancel{2} \times 3}{7 \times \cancel{2}} = \dfrac{12}{7}$.

Try this one:

What does the fraction $\dfrac{12}{16}$ reduce to?

(a) $\dfrac{2 \times 3 \times \cancel{2}}{8 \times \cancel{2}} = \dfrac{6}{8}$ Turn to page 164

(b) $\dfrac{\cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times 2 \times \cancel{2}} = \dfrac{3}{4}$ Turn to page 178

Correct. The fraction 11/8 does not reduce.

Try this one.

What does the fraction 12/16 reduce to?

(a) $\dfrac{2 \times 3 \times \cancel{2}}{8 \times \cancel{2}} = \dfrac{6}{8}$ Turn to page 186

(b) $\dfrac{\cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times 2 \times \cancel{2}} = \dfrac{3}{4}$ Turn to page 178

(c) $\dfrac{\cancel{2} \times 6}{\cancel{2} \times 4 \times 2} = \dfrac{6}{8}$ Turn to page 186

Very good. $\frac{2 \times 6}{8 \times 2} = \frac{\cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times 2 \times \cancel{2}} = \frac{3}{4}.$

Try this one.

18/12 reduces to:

(a) 2/3 Turn to page 194

(b) 9/6 Turn to page 194

(c) 3/2 Turn to page 170

Your answer was (a) change it to $\frac{2 \times 2}{4 \times 3}$. This is not quite correct.

Remember we want to change <u>all</u> the integers to primes. 4 is not a prime.

Factoring the integers in the fraction 6/9, we obtain:

(a) $\frac{3 \times 2}{3 \times 3}$ Turn to page 188

(b) $\frac{3 \times 2}{9}$ Turn to page 184

Excellent. We can say that either 17/14 doesn't reduce or simply write it as 17/14.

Remember in reducing fractions, we first factor the integers into their primes. Then we cancel as before and obtain our answer.

That is the end of this Section. Now turn to page 202, and we'll look at one last Section.

Incorrect. Remember, we want the fraction to be written in terms of prime numbers.

$\frac{8 \times 6}{18}$ can be factored into primes in the numerator

as $8 = 2 \times 2 \times 2$ in terms of primes

and $6 = 3 \times 2$ in terms of primes.

In the denominator (bottom) $18 = 9 \times 2$ which is

$3 \times 3 \times 2$.

Thus, we write the fraction as $\frac{(2 \times 2 \times 2) \times (3 \times 2)}{3 \times 3 \times 2}$.

Factoring the fraction 6/9, we first:

(a) change it to $\frac{6}{3 \times 3}$ Turn to page 164

(b) change it to $\frac{3 \times 2}{3 \times 3}$ Turn to page 173

Incorrect.

The fraction 18/12 = $\frac{3 \times 3 \times 2}{3 \times 2 \times 2}$.

You may cancel one pair of 3's and one pair of 2's like this: $\frac{\cancel{3} \times 3 \times \cancel{2}}{\cancel{3} \times 2 \times \cancel{2}}$ which gives us 3/2 for our answer.

Do you see where you made your mistake?

(a) Yes Turn to page 185

(b) No Turn to page 183

Incorrect. You are having trouble factoring integers into primes. Ask your human teacher for help, then return to the unit on factoring to primes, page 133.

Your answer was $\frac{3 \times 2}{9}$. But 9 is not a prime.

See your human teacher for help and then return to the unit on factoring integers to primes, page 133.

Excellent. Now you're on the right track.

Now that you can factor the numerator and denominator into primes, let's apply it to reducing fractions.

For example, on the fraction which you just factored into primes let's reduce it.

You said $12/8 = \frac{2 \times 3 \times 2}{2 \times 2 \times 2}$. Now, we can cancel as before. $\frac{\cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times \cancel{2}} = \frac{3}{2}$. Since $2/2 = 1$, we cancel them.

Try this one.

Reduce the fraction 14/28.

(a) $\frac{7 \times \cancel{2}}{14 \times \cancel{2}} = 7/14$ Turn to page 183

(b) $\frac{\cancel{7} \times \cancel{2}}{\cancel{7} \times 2 \times \cancel{2}} = 1/2$ Turn to page 189

Your answer was incorrect. Let's look at the problem again.

We want to reduce the fraction 12/16.

12 can be factored to 2 x 2 x 3.

16 can be factored to 2 x 2 x 2 x 2.

This will cancel as follows:

$\frac{\cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 2 \times 2}$ two pairs of 2's as 2/2 = 1.

This leaves us with $\frac{3}{2 \times 2}$ or 3/4.

We factor to primes, then cancel.

Reduce the following fraction: 26/10

(a) 13/5 Turn to page 192

(b) 2/5 Turn to page 193

Your answer was (a) $\frac{2 \times 2 \times 3}{2 \times 4}$. Incorrect.

What about the 4 in the denominator. It is not prime.

Did you make a careless mistake? Return to page 190

and look at the problem again.

Correct. 6/9 does factor to $\frac{3 \times 2}{3 \times 3}$.

Try another one.

Factor the integers in the fraction 12/8.

(a) $\frac{2 \times 2 \times 3}{2 \times 4}$ Turn to page 183

(b) $\frac{2 \times 3 \times 2}{2 \times 2 \times 2}$ Turn to page 185

Excellent. $14/28 = \frac{\cancel{7} \times \cancel{2}}{\cancel{7} \times \cancel{2} \times 2} = 1/2.$

Now turn to page 175.

Not quite. Your answer was $\frac{\cancel{2} \times 2}{\cancel{2} \times 2 \times 3} = 2/6$.

Now we can also cancel another pair of 2's since 2/2 = 1.

So $\frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 3} = 1/3$ not 2/6.

Factor 12/8 into primes.

(a) $\frac{2 \times 2 \times 3}{2 \times 4}$ Turn to page 187

(b) $\frac{2 \times 3 \times 2}{2 \times 2 \times 2}$ Turn to page 185

Your answer was incorrect.

It would be best if you returned to the beginning of this unit and started over. Be more careful this time.

Return to page 158.

Correct. $26/10 = \frac{13 \times \cancel{2}}{5 \times \cancel{2}} = 13/5.$

Try this one.

18/12 reduces to:

(a) 2/3	Turn to page 182
(b) 9/6	Turn to page 191
(c) 3/2	Turn to page 170

No. 26/10 reduces like this: $26/10 = \frac{13 \times \cancel{2}}{5 \times \cancel{2}} = 13/5.$

Does 15/6 reduce to 5/2?

(a) Yes Turn to page 198

(b) No Turn to page 191

No. $18/12 = \frac{\cancel{3} \times 3 \times \cancel{2}}{\cancel{3} \times 2 \times \cancel{2}} = 3/2.$

Be more careful this time.

Reduce the fraction 26/10.

(a) 5/13 Turn to page 191

(b) 13/5 Turn to page 192

(c) 2/5 Turn to page 193

Incorrect.

Look at the problem more carefully.

$15/6 = \frac{5 \times \cancel{3}}{2 \times \cancel{3}} = 5/2.$

Return to the beginning of this unit and see if you can't be more careful.

Turn to page 158.

Wait a minute. Look at 17/14 again.

17 is already prime and 14 = 7 x 2. Therefore, we cannot reduce this fraction.

Can the fraction 21/14 be reduced?

(a) Yes Turn to page 199

(b) No Turn to page 200

Your answer was incorrect. Look again.

$$21/14 = \frac{3 \times \cancel{7}}{2 \times \cancel{7}} = 3/2.$$

You are getting careless.

Does 15/6 reduce to 5/2?

(a) Yes Turn to page 198

(b) No Turn to page 191

Very good. $15/6 = \frac{5 \times \cancel{3}}{2 \times \cancel{3}} = 5/2.$

Try this one.

18/12 reduces to:

(a) 2/3 Turn to page 182

(b) 9/6 Turn to page 191

(c) 3/2 Turn to page 170

Your answer was (a) Yes. Correct.

What does 21/14 reduce to?

(a)	3/2	Turn to page 201
(b)	2/3	Turn to page 191
(c)	7/2	Turn to page 197

Your answer was (b) No. Incorrect.

Look again.

$$21/14 = \frac{3 \times \cancel{7}}{2 \times \cancel{7}} = 3/2.$$

You don't seem quite sure of reducing fractions yet. Return to the beginning of this unit, page 158, and begin again.

Correct. It does reduce to 3/2.

$$21/14 = \frac{\cancel{7} \times 3}{\cancel{7} \times 2} = 3/2.$$

That completes this Section. Now turn to page <u>202</u>,
and we'll look at one last Section.

Rather than reducing fractions, we can go in the reverse direction and get a fraction that has the same value but is not in its reduced form. Such a fraction is called an equivalent fraction.

One way to find an equivalent fraction of 1/2 is to take 1/2 x 2/2, and we will get 2/4 which has the same value since 2/2 = 1. We could also take 3/3 x 1/2 and get 3/6 since 3/3 also = 1. In other words, we can enlarge the numbers of any fraction by multiplying the fraction by an equivalent form of 1. Equivalent forms of 1 are 2/2, 3/3, 4/4, etc.

Question:

An equivalent fraction of 2/3 is:

(a)	4/6	Turn to page 208
(b)	6/9	Turn to page 208
(c)	2/6	Turn to page 205

Very good. However, did you notice that both 6/4 and 3/2 were correct answers? Notice that 2/2 x 6/4 = 12[illegible] [illegible]d that 4/4 x 3/2 = 12/8.

Question:

An equivalent fraction of 3/5 is:

(a) 21/35 Turn to page 207

(b) 6/15 Turn to page 209

(c) 12/15 Turn to page 211

Your answer of (a) Yes is correct.

5/4 can be multiplied by 4/4 to give 20/16.

$5/4 \times 4/4 = 20/16$.

An equivalent fraction of 3/5 is:

(a) 6/15 Turn to page 213

(b) 21/35 Turn to page 207

(c) 12/15 Turn to page 213

Incorrect. 2/6 is not equivalent to 2/3.

Remember, we can find an equivalent fraction for any other fraction by multiplying by 1. However, we write 1 in one of its equivalent forms, such as 2/2, 3/3, 4/4, etc.

Try this one.

An equivalent form of the fraction 1/4 is:

(a) 4/16 Turn to page 212

(b) 4/12 Turn to page 214

Your answer was incorrect.

12/8 is the equivalent fraction of 6/4 or 3/2.

This is because 6/4 x 2/2 = 12/8 and 3/2 x 4/4 = 12/8.

You are having trouble grasping this idea. Return to page 202 and reread the introduction. Then carefully do the problem again.

Excellent. 7/7 x 3/3 = 21/35.

Try this one.

What integer must you use in 2/5 = __/10 to make the fraction equivalent to 2/5?

(a) 2 Turn to page 217

(b) 5 Turn to page 219

(c) 4 Turn to page 216

Correct! 4/6 and 6/9 are both equivalent fractions of 2/3. You saw that 2/2 x 2/3 = 4/6 and that 3/3 x 2/3 = 6/9. Since 2/2 and 3/3 are equal to one, we haven't changed the value of the fraction.

Question:

12/8 is equivalent to which of the fraction(s) below?

(a) 6/4 Turn to page 215

(b) 4/3 Turn to page 210

(c) 3/2 Turn to page 203

No! 6/15 is not equivalent to 3/5 because 6/15 =

$\frac{2 \times 3}{3 \times 5}$ and the 2 does not cancel the 3.

Go to page 203 and look at the problem again.

Oops! Look again.

12/8 = 6/4 x 2/2 or 12/8 = 3/2 x 4/4.

Remember, we want to multiply by an equivalent form of 1, such as 2/2 or 3/3. Multiplication by 1 changes the form but not the value.

Question:

20/16 is equivalent to the fraction 5/4.

(a) Yes Turn to page 204

(b) No Turn to page 213

Wait a minute! Remember, we are multiplying by an equivalent form of 1.

Go to page 215 and look at the problem again.

Correct! 4/16 is an equivalent form of 1/4. You saw that this is done by multiplying 1/4 x 4/4 and getting 4/16.

Try this one.

12/8 is an equivalent form of which fraction(s) below?

(a) 6/4 Turn to page 203

(b) 4/3 Turn to page 206

(c) 3/2 Turn to page 215

Your last answer was incorrect. Return to page 202 and read the material carefully. Then continue from there.

No! An equivalent form of the fraction 1/4 is 4/16. Let's see why.

1/4 can be multiplied by 4/4 since 4/4 = 1, and anything multiplied by 1 equals itself. Therefore, 4/4 x 1/4 = 4/16.

Question:

20/16 is an equivalent form of the fraction 5/4.

(a) Yes Turn to page 204

(b) No Turn to page 213

Very good! However, did you notice that both 6/4 and 3/2 were correct answers? Notice that 2/2 x 6/4 = 12/8 and that 4/4 x 3/2 = 12/8.

Question:

An equivalent fraction of 3/5 is:

(a) 21/35 Turn to page 207

(b) 6/15 Turn to page 209

(c) 12/15 Turn to page 211

Very good! Let's try one more.

Question:

What integer in 3/5 = 18/__ will make the fraction equivalent to 3/5?

(a) 6 Turn to page 223

(b) 30 Turn to page 230

(c) 15 Turn to page 225

Incorrect.

What equivalent form of 1 can you use to change 2/5 into 2/10? There isn't any. Here's how you begin. $?/? \times 2/5 = \frac{}{10}$ is the problem, and we must find a number for the question mark that, when multiplied by 5, will give ten. That number is:

(a) 5 Turn to page 218

(b) 2 Turn to page 220

Wait a minute!

Where did you learn that 5 x 5 = 10?

Go back to page 217 and make another selection.

Wait a minute! Remember, we are multiplying by an equivalent form of 1.

Go back to page 207 and make another selection.

2 is the correct answer.

Now, ?/? x 2/5 = __/10 becomes 2/2 x 2/5 = __/10.

Question:

Which of the following integers must be placed into the __ to give us a fraction equivalent to 2/5?

(a) 2 Turn to page 222

(b) 4 Turn to page 216

(c) 5 Turn to page 227

What are you doing here? You must be playing games.

Go see your teacher and then come back to page 202.

Incorrect!

You are having trouble with your multiplication, aren't you?

2 x 2 = __.

(a) 4 Turn to page 216

(b) 2 Turn to page 221

Oops! Look again. You must have made a careless mistake.

Go back to page 216 and make another selection.

Good! You're on the right track. Let's finish the problem.

Question:

When we multiply 3/5 by 6/6, we get the equivalent fraction:

(a) 18/6 Turn to page 227

(b) 18/15 Turn to page 228

(c) 18/30 Turn to page 230

Incorrect!

Remember that we must multiply by the equivalent form of 1. 18/15 is not equal to 3/5 times any equivalent forms of 1.

Try another one.

What equivalent form of 1 must we multiply 3/5 by to get an equivalent fraction with a numerator of 18?

(a) 6/6 Turn to page 224

(b) 18/18 Turn to page 226

(c) 6/5 Turn to page 227

Wait a minute! 3 x 18 doesn't equal 18.

Go back to page 225 and be a little more careful in making your selections.

Your last answer was incorrect. Return to page 202 and read the material carefully. Then continue from there.

Whoops! We were a little careless there.

Now, I know that you know what 6/6 x 3/5 is. It is:

(a) 3/5 Turn to page 227

(b) 30/18 Turn to page 221

(c) 18/30 Turn to page 229

Good. Keep it up!

Question:

What integer in 5/8 = __/32 will make the fraction equivalent to 5/8?

(a) 20 Turn to page 230

(b) 4 Turn to page 225

(c) 8 Turn to page 213

Very good! You have shown that you understand the ideas of this unit. Let's review what we've done.

1. We have changed integers into equivalent forms.
2. We have reduced numerical fractions to equivalent forms.
3. We have reduced fractions containing letters to equivalent forms.
4. We have cancelled like numbers in the numerator and denominator.
5. We have defined prime numbers.
6. We have factored integers into prime factors.
7. We have reduced fractions by factoring and cancelling.
8. We have found equivalent forms of fractions.

Now, you should be ready for a test over this unit. If you need to review, refer to the following pages:

1 - 18	Equivalent forms of integers
19 - 47	Reducing fractions to equivalent forms using numbers

48 - 81	Reducing fractions to equivalent forms using letters
82 - 112	Cancelling
113 - 132	Prime numbers
133 - 157	Factoring integers into primes
158 - 201	Reducing fractions by factoring and cancellation
202 - 230	Equivalent fractions

Go tell your teacher that you have finished the unit.

TEST QUESTIONS

UNIT 3 - EQUIVALENT FORMS

Directions: The correct answers will always be expressed in lowest terms.

1. 10/2 is an equivalent form of the integer

(a) 10

(b) 5

(c) 12

2. If a = 10 and b = 5, then a/b is an equivalent form for

(a) 2

(b) 5

(c) 10

3. Which of the following sets are all "primes"?

(a) 2, 5, 7

(b) 3, 6, 9

(c) 5, 10, 15

4. 1 is a prime

(a) Yes

(b) No

5. When $\frac{8 \times 5}{8}$ is reduced by cancellation the answer is

(a) 1/5

(b) 8

(c) 5

6. Which of the following is an equivalent form of the integer 7?

(a) 7/3

(b) 14/3

(c) 21/3

7. $\frac{4}{2} = ?$

   (a) 1

   (b) 2

   (c) 3

8. Look at the following integers and determine which group contains all prime numbers.

   (a) 7, 13, 15, 19

   (b) 2, 17, 3, 5

   (c) 3, 13, 10, 5

9. The number 100 broken into primes is

   (a) 2 x 2 x 25

   (b) 4 x 5 x 5

   (c) 2 x 2 x 5 x 5

10. $\frac{5 \times 13}{9}$ reduces to

   (a) 65

   (b) $\frac{5 \times \cancel{10}^{3}}{\cancel{9}\ 3} = 5$

   (c) Doesn't cancel or reduce

11. The fraction 10/5 is an equivalent form of __?__

   (a) 2

   (b) 10

   (c) 5

12. a/a is the equivalent form of __?__

   (a) a

   (b) 2

   (c) 1

13. Can the integer 13 be factored into primes?

    (a) Yes

    (b) No

14. $\frac{4 \times 11}{11} = ?$

    (a) 11

    (b) 4

    (c) 1

15. $\frac{18}{12} = ?$

    (a) 3/2

    (b) 9/6

    (c) 3/4

16. Which of the following are equivalent forms of the integer 5

    (a) 1/5, 10

    (b) 5/1, 10/2

    (c) Both a and b

17. $\frac{5}{5} = ?$

    (a) 5

    (b) 2

    (c) 1

18. Can the number 18 be factored into primes?

    (a) Yes

    (b) No

19. Reduce the fraction 26/10

(a) 5/13

(b) 2/5

(c) 13/5

20. 20/14 is an equivalent form of the fraction 10/7

(a) Yes

(b) No

21. C/C is an equivalent form of the integer

(a) 2

(b) 1

(c) 6

22. R/R =

(a) 2

(b) R

(c) 1

23. The prime factors of the number 12 are?

(a) 4 x 3

(b) 6 x 2

(c) 2 x 2 x 3

24. The fraction 3/5 is equivalent to the fraction

(a) 15/25

(b) 10/9

(c) 9/25

Answer Sheet - Unit 3

Objective	Question Number	Answer
1	1	b
2	2	a
3	3	a
4	4	b
2, 5	5	c
1	6	c
2	7	b
3	8	a
4	9	c
5	10	c
1	11	a
2	12	c
3	13	b
5	14	b
5	15	a
1	16	b
2	17	c
3	18	a
5	19	c
2, 5	20	a
1	21	b
2	22	c
3	23	c
2, 5	24	a
5	25	c

Objective	Questions
1	1, 6, 11, 16, 21
2	2, 5, 7, 12, 17, 20, 22, 24
3	3, 8, 13, 18, 23
4	4, 9
5	5, 10, 14, 15, 19, 20, 24, 25

OE 6000 (REV. 9-66)

DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
OFFICE OF EDUCATION

ERIC REPORT RESUME

ERIC ACCESSION NO.

CLEARINGHOUSE ACCESSION NUMBER | RESUME DATE June 1968 | P.A. | T.A.

IS DOCUMENT COPYRIGHTED? YES ☐ NO ☒
ERIC REPRODUCTION RELEASE? YES ☒ NO ☐

TITLE

Occupational Mathematics - EQUIVALENT FORMS

PERSONAL AUTHORS

Rahmlow, Harold; Ostheller, Karl; Potratz, Clarence; Winchell, L.; Snoey, Arthur

INSTITUTION SOURCE

Washington State University, Pullman, Washington, Dept. of Education

SOURCE CODE

REPORT/SERIES NO. Final Report No. 16-C

OTHER SOURCE

SOURCE CODE

OTHER REPORT NO.

OTHER SOURCE

SOURCE CODE

OTHER REPORT NO.

PUB'L. DATE June 1968 -- | CONTRACT GRANT NUMBER OEG-4-7-070031-1626

PAGINATION, ETC.

234 pages

RETRIEVAL TERMS

Occupational mathematics
Programmed instruction
Reduction of fractions

IDENTIFIERS

Vo-Tech. Education Research and Development Project (no. 7-0031)

ABSTRACT

One book of a 21-book series of programmed instruction materials designed to help pupils acquire mathematics capabilities most useful in sub-professional level occupations. Other programmed books in the series are:

- Symbols
- Representing Numbers by Letters
- Ratios and Fractions
- Addition of Fractions
- Subtraction of Fractions
- Multiplication of Fractions
- Division of Fractions
- Concepts of Decimals and Fractions
- Addition and Subtraction of Decimals
- Multiplication of Decimals
- Division of Decimals
- Conversion of Fractions into Decimals
- Equivalent Forms of A = BC
- Solutions of A = BC
- Percentage
- Commutative Law
- Reciprocals
- Scientific Notation
- Proportions
- Concepts of Number Bases